1816 $\int \frac{x^2}{\left(x^2+1\right)^2}dx=-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\operatorname{arctg}x+C$

1869

$$\int \frac{x^3+1}{x^3-5x^2+6x}dx$$

$$\frac{x^3+1}{x^3-5x^2+6x}=1+\frac{5x^2-6x+1}{x^3-5x^2+6x}$$

$$\frac{5x^2-6x+1}{x^3-5x^2+6x}=\frac{5x^2-6x+1}{x\left(x-2\right)\left(x-3\right)}=\frac{A}{x}+\frac{B}{x-2}+\frac{C}{x-3}$$

$$5x^2-6x+1=A\left(x-2\right)\left(x-3\right)+Bx\left(x-3\right)+Cx\left(x-2\right)$$

$$x=0:1=6A,\ A=\frac{1}{6}$$

$$x=2:\ 9=-2B,\ \ B=-\frac{9}{2}$$

$$x=3:28=3C,\ C=\frac{28}{3}$$

$$\int \frac{x^3+1}{x^3-5x^2+6x}dx=x+\frac{1}{6}\ln\left|x\right|-\frac{9}{2}\ln\left|x-2\right|+\frac{28}{3}\ln\left|x-3\right|+C$$

1882 $\int \frac{x}{x^3-1}dx=\frac{1}{3}\int\left(\frac{1}{x-1}-\frac{x-1}{x^2+x+1}\right)dx=\frac{1}{6}\ln\frac{\left(x-1\right)^2}{x^2+x+1}+\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}}+C$

$$\int \frac{x}{\left(x^3-1\right)^2}dx=\int \frac{x-x^4+x^4}{\left(x^3-1\right)^2}dx=-\int \frac{x}{x^3-1}dx-\frac{1}{3}\int x^2d\,\frac{1}{x^3-1}=$$

$$=-\int \frac{x}{x^3-1}dx-\frac{1}{3}\frac{x^2}{x^3-1}+\frac{2}{3}\int \frac{x}{x^3-1}dx=-\frac{1}{3}\frac{x^2}{x^3-1}-\frac{1}{3}\int \frac{x}{x^3-1}dx=$$

$$=-\frac{1}{3}\frac{x^2}{x^3-1}-\frac{1}{3}\left(\frac{1}{6}\ln\frac{\left(x-1\right)^2}{x^2+x+1}+\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}}\right)+C=$$

$$=-\frac{1}{3}\frac{x^2}{x^3-1}-\frac{1}{18}\ln\frac{\left(x-1\right)^2}{x^2+x+1}-\frac{1}{3\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}}+C$$

1886

$$\int \frac{dx}{x^6+1}=\frac{1}{3}\int\left(\frac{1}{x^2+1}-\frac{x^2-2}{x^4-x^2+1}\right)dx$$

$$\frac{x^2-2}{x^4-x^2+1}=\frac{Ax+B}{x^2-\sqrt{3}x+1}+\frac{Cx+D}{x^2+\sqrt{3}x+1}$$

$$x^2-2=(Ax+B)\left(x^2+\sqrt{3}x+1\right)+(Cx+D)\left(x^2-\sqrt{3}x+1\right)$$

$$\begin{cases} A+C=0,\ C=-A \\ \sqrt{3}A+B-\sqrt{3}C+D=1,\quad A=\dfrac{\sqrt{3}}{2},\ C=-\dfrac{\sqrt{3}}{2} \\ A+\sqrt{3}B+C-\sqrt{3}D=0,\ B=D \\ B+D=-2,\ B=D=-1 \end{cases}$$

$$\int\frac{x^2-2}{x^4-x^2+1}dx=\frac{\sqrt{3}}{2}\int\left(\frac{x-\frac{2}{\sqrt{3}}}{x^2-\sqrt{3}x+1}-\frac{x+\frac{2}{\sqrt{3}}}{x^2+\sqrt{3}x+1}\right)dx=$$

$$=\frac{\sqrt{3}}{2}\int\left(\frac{\left(x-\frac{\sqrt{3}}{2}\right)-\frac{1}{2\sqrt{3}}}{\left(x-\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4}}-\frac{\left(x+\frac{\sqrt{3}}{2}\right)+\frac{1}{2\sqrt{3}}}{\left(x+\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4}}\right)dx=$$

$$=\frac{\sqrt{3}}{4}\ln\frac{x^2-\sqrt{3}x+1}{x^2+\sqrt{3}x+1}-\frac{1}{2}\operatorname{arctg}\left(2x-\sqrt{3}\right)-\frac{1}{2}\operatorname{arctg}\left(2x+\sqrt{3}\right)+C$$

$$\int\frac{dx}{x^6+1}=\frac{1}{3}\int\left(\frac{1}{x^2+1}-\frac{x^2-2}{x^4-x^2+1}\right)dx=$$

$$=\frac{1}{3}\operatorname{arctg}x-\frac{1}{4\sqrt{3}}\ln\frac{x^2-\sqrt{3}x+1}{x^2+\sqrt{3}x+1}+\frac{1}{6}\operatorname{arctg}\left(2x-\sqrt{3}\right)+\frac{1}{6}\operatorname{arctg}\left(2x+\sqrt{3}\right)+C$$

или

$$\int\frac{dx}{x^6+1}=\frac{1}{2}\int\frac{\left(x^4-x^2+1\right)-\left(x^4-1\right)+x^2}{x^6+1}dx$$

$$\int\frac{x^4-x^2+1}{x^6+1}dx=\int\frac{dx}{x^2+1}=\operatorname{arctg}x+C$$

$$\int \frac{x^4-1}{x^6+1}dx = \int \frac{x^2-1}{x^4-x^2+1}dx = \int \frac{1-\frac{1}{x^2}}{x^2-1+\frac{1}{x^2}}dx = \left[y = x+\frac{1}{x} \right] =$$

$$\int \frac{dy}{y^2-3} = \frac{1}{2\sqrt{3}}\ln\frac{y-\sqrt{3}}{y+\sqrt{3}} + C = \frac{1}{2\sqrt{3}}\ln\frac{x^2-\sqrt{3}x+1}{x^2+\sqrt{3}x+1} + C$$

$$\int \frac{x^2}{x^6+1}dx = \frac{1}{3}\operatorname{arctg} x^3 + C$$

$$\int \frac{dx}{x^6+1} = \frac{1}{2}\operatorname{arctg} x + \frac{1}{6}\operatorname{arctg} x^3 + \frac{1}{4\sqrt{3}}\ln\frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} + C$$

1906

$$\int \frac{x^2+x}{x^6+1}dx$$

$$\int \frac{x^2}{x^6+1}dx = \frac{1}{3}\operatorname{arctg} x^3 + C$$

$$\int \frac{x}{x^6+1}dx = \left[y = x^2 \right] = \frac{1}{2}\int \frac{dy}{y^3+1} = \frac{1}{6}\int \left(\frac{1}{y+1} - \frac{y-2}{y^2-y+1} \right) dy =$$

$$= \frac{1}{12}\ln\frac{(y+1)^2}{y^2-y+1} + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2y-1}{\sqrt{3}} + C$$

$$\int \frac{x^2+x}{x^6+1}dx = \frac{1}{3}\operatorname{arctg} x^3 + \frac{1}{12}\ln\frac{(x^2+1)^2}{x^4-x^2+1} + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2x^2-1}{\sqrt{3}} + C$$