

Найдите $dz, d^2z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial x \partial y}, \frac{\partial^2 z}{\partial y^2}$ в точке $(1, 1, 2)$, если

$$x^2z + yz + xyz^3 = 12.$$

$$2xzdx + x^2dz + zdy + ydz + yz^3dx + xz^3dy + 3xyz^2dz = 0$$

$$(1, 1, 2): 4dx + dz + 2dy + dz + 8dx + 8dy + 12dz = 0$$

$$12dx + 10dy + 14dz = 0, \quad dz = -\frac{6}{7}dx - \frac{5}{7}dy; \quad \frac{\partial z}{\partial x} = -\frac{6}{7}, \frac{\partial z}{\partial y} = -\frac{5}{7};$$

$$2zdx^2 + 4xdzdx + 2dydz + 2z^3dxdy + 6yz^2dxdz + 6xz^2dydz + 6xyzdz^2 + (x^2 + y + 3xyz^2)d^2z = 0$$

$$(1, 1, 2): 4dx^2 + 4dzdx + 2dydz + 16dxdy + 24dxdz + 24dydz + 12dz^2 + 14d^2z = 0$$

$$4dx^2 + 16dxdy + 28dxdz + 26dydz + 12dz^2 + 14d^2z = 0$$

$$2dx^2 + 8dxdy + 14dxdz + 13dydz + 6dz^2 + 7d^2z = 0$$

$$2dx^2 + 8dxdy + dz(14dx + 13dy + 6dz) + 7d^2z = 0$$

$$2dx^2 + 8dxdy + \left(-\frac{6}{7}dx - \frac{5}{7}dy\right)\left(14dx + 13dy + 6\left(-\frac{6}{7}dx - \frac{5}{7}dy\right)\right) + 7d^2z = 0$$

$$2dx^2 + 8dxdy - \left(\frac{6}{7}dx + \frac{5}{7}dy\right)\left(\frac{62}{7}dx + \frac{61}{7}dy\right) + 7d^2z = 0$$

$$-\frac{274}{49}dx^2 - \frac{284}{49}dxdy - \frac{305}{49}dy^2 + 7d^2z = 0$$

$$d^2z = \frac{274}{343}dx^2 + \frac{284}{343}dxdy + \frac{305}{343}dy^2$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{274}{343}, \frac{\partial^2 z}{\partial x \partial y} = \frac{142}{343}, \frac{\partial^2 z}{\partial y^2} = \frac{305}{343}$$

3395. Найдите $\frac{\partial^2 z}{\partial x \partial y}$, если $F(x + y + z, x^2 + y^2 + z^2) = 0$.

$$F(x + y + z, x^2 + y^2 + z^2) = 0$$

$$F_1' \left(1 + \frac{\partial z}{\partial x} \right) + F_2' \left(2x + 2z \frac{\partial z}{\partial x} \right) = 0,$$

$$\frac{\partial z}{\partial x} = -\frac{F_1' + F_2' 2x}{F_1' + F_2' 2z}, 1 + \frac{\partial z}{\partial x} = \frac{2F_2'(z - x)}{F_1' + F_2' 2z}; 2x + 2z \frac{\partial z}{\partial x} = \frac{2F_1'(x - z)}{F_1' + F_2' 2z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_1' + F_2' 2y}{F_1' + F_2' 2z}, 1 + \frac{\partial z}{\partial y} = \frac{2F_2'(z - y)}{F_1' + F_2' 2z}; 2y + 2z \frac{\partial z}{\partial y} = \frac{2F_1'(y - z)}{F_1' + F_2' 2z};$$

$$F_{11}'' \left(1 + \frac{\partial z}{\partial x} \right) \left(1 + \frac{\partial z}{\partial y} \right) + F_{12}'' \left(1 + \frac{\partial z}{\partial x} \right) \left(2y + 2z \frac{\partial z}{\partial y} \right) + F_{12}'' \left(1 + \frac{\partial z}{\partial y} \right) \left(2x + 2z \frac{\partial z}{\partial x} \right) +$$

$$+ F_{22}'' \left(2x + 2z \frac{\partial z}{\partial x} \right) \left(2y + 2z \frac{\partial z}{\partial y} \right) + F_1' \frac{\partial^2 z}{\partial x \partial y} + 2F_2' \left(\frac{\partial z}{\partial x} \frac{\partial z}{\partial y} + z \frac{\partial^2 z}{\partial x \partial y} \right) = 0;$$

$$F_{11}'' \frac{2F_2'(z - x)}{F_1' + F_2' 2z} \frac{2F_2'(z - y)}{F_1' + F_2' 2z} + F_{12}'' \frac{2F_2'(z - x)}{F_1' + F_2' 2z} \frac{2F_1'(y - z)}{F_1' + F_2' 2z} + F_{12}'' \frac{2F_2'(z - y)}{F_1' + F_2' 2z} \frac{2F_1'(x - z)}{F_1' + F_2' 2z} +$$

$$+ F_{22}'' \frac{2F_1'(x - z)}{F_1' + F_2' 2z} \frac{2F_1'(y - z)}{F_1' + F_2' 2z} + F_1' \frac{\partial^2 z}{\partial x \partial y} + 2F_2' \left(\frac{\partial z}{\partial x} \frac{\partial z}{\partial y} + z \frac{\partial^2 z}{\partial x \partial y} \right) = 0;$$

$$\frac{4(z - x)(z - y)(F_{11}'' F_2'^2 - 2F_{12}'' F_1' F_2' + F_{22}'' F_1'^2)}{(F_1' + F_2' 2z)^2} + 2F_2' \frac{\partial z}{\partial x} \frac{\partial z}{\partial y} + (F_1' + F_2' 2z) \frac{\partial^2 z}{\partial x \partial y} = 0;$$

$$\frac{4(z - x)(z - y)(F_{11}'' F_2'^2 - 2F_{12}'' F_1' F_2' + F_{22}'' F_1'^2)}{(F_1' + F_2' 2z)^2} + 2F_2' \frac{F_1' + F_2' 2x}{F_1' + F_2' 2z} \frac{F_1' + F_2' 2y}{F_1' + F_2' 2z} + (F_1' + F_2' 2z) \frac{\partial^2 z}{\partial x \partial y} = 0;$$

$$\frac{4(z - x)(z - y)(F_{11}'' F_2'^2 - 2F_{12}'' F_1' F_2' + F_{22}'' F_1'^2) + 2F_2'(F_1' + F_2' 2x)(F_1' + F_2' 2y)}{(F_1' + F_2' 2z)^2} + (F_1' + F_2' 2z) \frac{\partial^2 z}{\partial x \partial y} = 0;$$

$$\frac{\partial^2 z}{\partial x \partial y} = -\frac{4(z - x)(z - y)(F_{11}'' F_2'^2 - 2F_{12}'' F_1' F_2' + F_{22}'' F_1'^2) + 2F_2'(F_1' + F_2' 2x)(F_1' + F_2' 2y)}{(F_1' + F_2' 2z)^3}$$

3405. Найдите du, dv, d^2u, d^2v при $x=1, y=1, u=0, v=\frac{\pi}{4}$, если

$$U = \frac{u}{x}, V = \frac{v}{y}; \quad U=0, V=\frac{\pi}{4}$$

$$e^U \cos V = \frac{x}{\sqrt{2}}, e^U \sin V = \frac{y}{\sqrt{2}}$$

$$\begin{cases} e^U \cos V dU - e^U \sin V dV = \frac{dx}{\sqrt{2}} \\ e^U \sin V dU + e^U \cos V dV = \frac{dy}{\sqrt{2}} \end{cases}$$

$$\begin{cases} e^U \cos V dU^2 - 2e^U \sin V dU dV - e^U \cos V dV^2 + e^U \cos V d^2U - e^U \sin V d^2V = 0 \\ e^U \sin V dU^2 + 2e^U \cos V dU dV - e^U \sin V dV^2 + e^U \sin V d^2U + e^U \cos V d^2V = 0 \end{cases}$$

Для выбранной точки

$$\begin{cases} dU - dV = dx \\ dU + dV = dy \end{cases} \begin{cases} dU = \frac{dx + dy}{2} \\ dV = \frac{dy - dx}{2} \end{cases}$$

$$\begin{cases} dU^2 - 2dU dV - dV^2 + d^2U - d^2V = 0 \\ dU^2 + 2dU dV - dV^2 + d^2U + d^2V = 0 \end{cases} \begin{cases} dU^2 - dV^2 + d^2U = 0 \\ 2dU dV + d^2V = 0 \end{cases} \begin{cases} d^2U = -dxdy \\ d^2V = \frac{dx^2 - dy^2}{2} \end{cases}$$

$$u = xU, \quad du = Udx + x dU, \quad d^2u = 2dx dU + x d^2U$$

$$v = yV, \quad dv = Vdy + y dV, \quad d^2v = 2dy dV + y d^2V$$

Для выбранной точки

$$du = dU = \frac{dx + dy}{2},$$

$$d^2u = 2dx dU + d^2U = dx(dx + dy) - dxdy = dx^2$$

$$dv = \frac{\pi}{4} dy + \frac{-dx + dy}{2}$$

$$d^2v = 2dy dV + d^2V = dy(dy - dx) + \frac{dx^2 - dy^2}{2} = \frac{(dx - dy)^2}{2}$$