Найдите $dz, d^2z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial x \partial y}, \frac{\partial^2 z}{\partial y^2}$ в точке $(1,1,2)$, если

$$x^2z + yz + xyz^3 = 12 .$$

$$2xzdx + x^2dz + zdy + ydz + yz^3dx + xz^3dy + 3xyz^2dz = 0$$

$$(1,1,2):\ 4dx + dz + 2dy + dz + 8dx + 8dy + 12dz = 0$$

$$12dx + 10dy + 14dz = 0,\ \ dz = -\frac{6}{7}dx - \frac{5}{7}dy; \quad \frac{\partial z}{\partial x} = -\frac{6}{7}, \frac{\partial z}{\partial y} = -\frac{5}{7};$$

$$2zdx^2 + 4xdzdx + 2dydz + 2z^3dxdy + 6yz^2dxdz + 6xz^2dydz + 6xyzdz^2 + \left(x^2 + y + 3xyz^2\right)d^2z = 0$$

$$(1,1,2):\ \ 4dx^2 + 4dzdx + 2dydz + 16dxdy + 24dxdz + 24dydz + 12dz^2 + 14d^2z = 0$$

$$4dx^2 + 16dxdy + 28dxdz + 26dydz + 12dz^2 + 14d^2z = 0$$

$$2dx^2 + 8dxdy + 14dxdz + 13dydz + 6dz^2 + 7d^2z = 0$$

$$2dx^2 + 8dxdy + dz\left(14dx + 13dy + 6dz\right) + 7d^2z = 0$$

$$2dx^2 + 8dxdy + \left(-\frac{6}{7}dx - \frac{5}{7}dy\right)\left(14dx + 13dy + 6\left(-\frac{6}{7}dx - \frac{5}{7}dy\right)\right) + 7d^2z = 0$$

$$2dx^2 + 8dxdy - \left(\frac{6}{7}dx + \frac{5}{7}dy\right)\left(\frac{62}{7}dx + \frac{61}{7}dy\right) + 7d^2z = 0$$

$$-\frac{274}{49}dx^2 - \frac{284}{49}dxdy - \frac{305}{49}dy^2 + 7d^2z = 0$$

$$d^2z = \frac{274}{343}dx^2 + \frac{284}{343}dxdy + \frac{305}{343}dy^2$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{274}{343}, \frac{\partial^2 z}{\partial x \partial y} = \frac{142}{343}, \frac{\partial^2 z}{\partial y^2} = \frac{305}{343}$$

3395. Найдите $\dfrac{\partial^2 z}{\partial x \partial y}$, если $F\left(x+y+z, x^2+y^2+z^2\right)=0$.

$$F\left(x+y+z, x^2+y^2+z^2\right)=0$$

$$F_1'\left(1+\frac{\partial z}{\partial x}\right)+F_2'\left(2x+2z\frac{\partial z}{\partial x}\right)=0,$$

$$\frac{\partial z}{\partial x}=-\frac{F_1'+F_2'2x}{F_1'+F_2'2z}, 1+\frac{\partial z}{\partial x}=\frac{2F_2'\left(z-x\right)}{F_1'+F_2'2z}; 2x+2z\frac{\partial z}{\partial x}=\frac{2F_1'\left(x-z\right)}{F_1'+F_2'2z}$$

$$\frac{\partial z}{\partial y}=-\frac{F_1'+F_2'2y}{F_1'+F_2'2z}; 1+\frac{\partial z}{\partial y}=\frac{2F_2'\left(z-y\right)}{F_1'+F_2'2z}; 2y+2z\frac{\partial z}{\partial y}=\frac{2F_1'\left(y-z\right)}{F_1'+F_2'2z};$$

$$F_{11}''\left(1+\frac{\partial z}{\partial x}\right)\left(1+\frac{\partial z}{\partial y}\right)+F_{12}''\left(1+\frac{\partial z}{\partial x}\right)\left(2y+2z\frac{\partial z}{\partial y}\right)+F_{12}''\left(1+\frac{\partial z}{\partial y}\right)\left(2x+2z\frac{\partial z}{\partial x}\right)+$$

$$+F_{22}''\left(2x+2z\frac{\partial z}{\partial x}\right)\left(2y+2z\frac{\partial z}{\partial y}\right)+F_1'\frac{\partial^2 z}{\partial x\partial y}+2F_2'\left(\frac{\partial z}{\partial x}\frac{\partial z}{\partial y}+z\frac{\partial^2 z}{\partial x\partial y}\right)=0;$$

$$F_{11}''\frac{2F_2'\left(z-x\right)}{F_1'+F_2'2z}\frac{2F_2'\left(z-y\right)}{F_1'+F_2'2z}+F_{12}''\frac{2F_2'\left(z-x\right)}{F_1'+F_2'2z}\frac{2F_1'\left(y-z\right)}{F_1'+F_2'2z}+F_{12}''\frac{2F_2'\left(z-y\right)}{F_1'+F_2'2z}\frac{2F_1'\left(x-z\right)}{F_1'+F_2'2z}+$$

$$+F_{22}''\frac{2F_1'\left(x-z\right)}{F_1'+F_2'2z}\frac{2F_1'\left(y-z\right)}{F_1'+F_2'2z}+F_1'\frac{\partial^2 z}{\partial x\partial y}+2F_2'\left(\frac{\partial z}{\partial x}\frac{\partial z}{\partial y}+z\frac{\partial^2 z}{\partial x\partial y}\right)=0;$$

$$\frac{4\left(z-x\right)\left(z-y\right)\left(F_{11}''F_2'^2-2F_{12}''F_1'F_2'+F_{22}''F_1'^2\right)}{\left(F_1'+F_2'2z\right)^2}+2F_2'\frac{\partial z}{\partial x}\frac{\partial z}{\partial y}+\left(F_1'+F_2'2z\right)\frac{\partial^2 z}{\partial x\partial y}=0;$$

$$\frac{4\left(z-x\right)\left(z-y\right)\left(F_{11}''F_2'^2-2F_{12}''F_1'F_2'+F_{22}''F_1'^2\right)}{\left(F_1'+F_2'2z\right)^2}+2F_2'\frac{F_1'+F_2'2x}{F_1'+F_2'2z}\frac{F_1'+F_2'2y}{F_1'+F_2'2z}+\left(F_1'+F_2'2z\right)\frac{\partial^2 z}{\partial x\partial y}=0;$$

$$\frac{4\left(z-x\right)\left(z-y\right)\left(F_{11}''F_2'^2-2F_{12}''F_1'F_2'+F_{22}''F_1'^2\right)+2F_2'\left(F_1'+F_2'2x\right)\left(F_1'+F_2'2y\right)}{\left(F_1'+F_2'2z\right)^2}+\left(F_1'+F_2'2z\right)\frac{\partial^2 z}{\partial x\partial y}=0;$$

$$\frac{\partial^2 z}{\partial x\partial y}=-\frac{4\left(z-x\right)\left(z-y\right)\left(F_{11}''F_2'^2-2F_{12}''F_1'F_2'+F_{22}''F_1'^2\right)+2F_2'\left(F_1'+F_2'2x\right)\left(F_1'+F_2'2y\right)}{\left(F_1'+F_2'2z\right)^3}$$

3405. Найдите du, dv, d^2u, d^2v при $x=1, y=1, u=0, v=\frac{\pi}{4}$, если

$$U=\frac{u}{x}, V=\frac{v}{y};\quad U=0, V=\frac{\pi}{4}$$

$$e^U\cos V=\frac{x}{\sqrt{2}}, e^U\cos V=\frac{y}{\sqrt{2}}$$

$$\begin{cases} e^U\cos V dU - e^U\sin V dV=\frac{dx}{\sqrt{2}} \\ e^U\sin V dU + e^U\cos V dV=\frac{dy}{\sqrt{2}} \end{cases}$$

$$\begin{cases} e^U\cos V dU^2 - 2e^U\sin V dU dV - e^U\cos V dV^2 + e^U\cos V d^2U - e^U\sin V d^2V=0 \\ e^U\sin V dU^2 + 2e^U\cos V dU dV - e^U\sin V dV^2 + e^U\sin V d^2U + e^U\cos V d^2V=0 \end{cases}$$

Для выбранной точки

$$\begin{cases} dU-dV=dx \\ dU+dV=dy \end{cases} \quad \begin{cases} dU=\frac{dx+dy}{2} \\ dV=\frac{dy-dx}{2} \end{cases}$$

$$\begin{cases} dU^2-2dUdV-dV^2+d^2U-d^2V=0 \\ dU^2+2dUdV-dV^2+d^2U+d^2V=0 \end{cases} \quad \begin{cases} dU^2-dV^2+d^2U=0 \\ 2dUdV+d^2V=0 \end{cases} \quad \begin{cases} d^2U=-dxdy \\ d^2V=\frac{dx^2-dy^2}{2} \end{cases}$$

$$u=xU,\ du=Udx+xdU, d^2U=2dxdU+xd^2U$$
$$v=yV,\ dv=Vdy+ydV, d^2V=2dydV+yd^2V$$

Для выбранной точки

$$du=dU=\frac{dx+dy}{2},$$
$$d^2U=2dxdU+d^2U=dx\left(dx+dy\right)-dxdy=dx^2$$

$$dv=\frac{\pi}{4}dy+\frac{-dx+dy}{2}$$
$$d^2V=2dydV+d^2V=dy\left(dy-dx\right)+\frac{dx^2-dy^2}{2}=\frac{\left(dx-dy\right)^2}{2}$$