Дифференциал функции

1087 $y=\frac{1}{2a}\ln\frac{x-a}{x+a}$

$$dy=\frac{1}{2a}\frac{x+a}{x-a}\frac{\left(x+a\right)dx-\left(x-a\right)dx}{\left(x+a\right)^{2}}=\frac{1}{2a}\frac{2adx}{x^{2}-a^{2}}=\frac{dx}{x^{2}-a^{2}}$$

1095 $y=\ln\sqrt{u^{2}+v^{2}}$

1100 $\sin 29°$

$y=\sin x,$

$dy=\cos xdx;$

$x_{0}=\pi/6,\ y_{0}=\frac{1}{2};$

$dy=\frac{\sqrt{3}}{2}dx,\ dx=-\frac{\pi}{180},\ dy=-\frac{\sqrt{3}}{2}\frac{\pi}{180}\approx-0.01512;$

$\Delta=\sin 29°-\sin 30°\approx-0.01519$

Производные и дифференциалы высших порядков

$$f''(x_0) = (f')'(x_0)$$

$$f^{(n)}(x_0) = \left(f^{(n-1)}\right)'(x_0)$$

$$f',\ f'',\ f''',\ f^{IV},\ f^{V},\ f^{VI}$$

$$dy = f'(x)dx,\ d^2y = f''(x)dx^2,\ d^3y = f'''(x)dx^3, \ldots$$

$$(f \cdot g)^{(n)} = \sum_{k=0}^{n} C_n^k f^{(n-k)} g^{(k)}$$

. Таблица производных

$$\left(x^{\mu}\right)^{(n)} = \mu(\mu-1)\cdots(\mu-n+1)x^{\mu-n+1}$$

$$\left(e^x\right)^{(n)} = e^x$$

$$(\sin x)^{(n)} = \sin\left(x + \frac{\pi n}{2}\right),$$

$$(\cos x)^{(n)} = \cos\left(x + \frac{\pi n}{2}\right)$$

Найдите y''

1112 $y = \dfrac{x}{\sqrt{1-x^2}}$

$$y' = \frac{\sqrt{1-x^2} + \dfrac{x^2}{\sqrt{1-x^2}}}{1-x^2} = \frac{1}{\left(1-x^2\right)^{3/2}},\ y'' = \frac{3x}{\left(1-x^2\right)^{3/2}}$$

1116 $y = \dfrac{\arcsin x}{\sqrt{1-x^2}}$

1122 $y = \ln\dfrac{u}{v}$

1124 $y = u^v$

$$y = e^{v \ln u};$$

$$y' = e^{v \ln u}\left(v' \ln u + \frac{vu'}{u}\right)$$

$$y'' = u^{v}\left(v' \ln u + \frac{vu'}{u}\right)^{2} + u^{v}\left(v'' \ln u + 2\frac{v'u'}{u} - \frac{vu'^{2}}{u^{2}} + \frac{vu''}{u}\right)$$

1126 Найдите y''':

$$y = f\left(\frac{1}{x}\right)$$

$$y' = -f' \cdot \frac{1}{x^{2}},\ y'' = f'' \cdot \frac{1}{x^{4}} + f' \cdot \frac{2}{x^{3}},\ y''' = -f''' \cdot \frac{1}{x^{6}} - f'' \cdot \frac{6}{x^{5}} - f' \frac{6}{x^{5}}$$

Найдите $d^2 y$:

1135

$$y = \frac{u}{v}$$

1139

$$y = \operatorname{arctg}\frac{u}{v}$$

$$dy = \frac{1}{1 + \frac{u^{2}}{v^{2}}}\frac{vdu - udv}{v^{2}} = \frac{vdu - udv}{u^{2} + v^{2}},\ d^{2}y = \frac{vd^{2}u - ud^{2}v}{u^{2} + v^{2}} - \frac{2(vdu - udv)(udu + vdv)}{(u^{2} + v^{2})^{2}}.$$

Найдите производные и дифференциалы указанных порядков

1159 $y = \dfrac{x^{2}}{1 - x};\quad y^{(8)}$

1162 $y = \dfrac{e^{x}}{x};\quad y^{(10)}$

1167 $y = \sin x \sin 2x \sin 3x \quad y^{(10)}$

$$y = \frac{1}{2}(\cos x - \cos 3x)\sin 3x = \frac{1}{4}(\sin 2x + \sin 4x - \sin 6x),$$

$$y^{(10)} = \frac{1}{4}\left(-2^{10}\sin 2x - 4^{10}\sin 4x + 6^{10}\sin 6x\right).$$

1169 $y = e^x \cos x \qquad y^{IV}$

$$y' = e^x \cos x - e^x \sin x,\ y'' = e^x \cos x - e^x \sin x - e^x \sin x - e^x \cos x = -2e^x \sin x,$$
$$y''' = -2e^x \sin x - 2e^x \cos x,\ y^{IV} = -4e^x \cos x.$$

1177 $y = e^u \qquad d^4 y$

$$dy = e^u du,\ d^2 y = e^u du^2 + e^u d^2 u,$$
$$d^3 y = e^u du^3 + 3e^u du d^2 u + e^u d^3 u,$$
$$d^4 y = e^u du^4 + 6e^u du^2 d^2 u + 3e^u \left(d^2 u\right)^2 + 4e^u du d^3 u + e^u d^4 u$$

Найдите $y^{(n)}$

1193 $y = \sin^2 x$

1195 $y = \sin^3 x$

1204 $y = \left(x^2 + 2x + 2\right)e^{-x}$

$$y^{(n)} = (-1)^n \left(x^2 + 2x + 2\right)e^{-x} + (-1)^{n-1} C_n^1 \left(2x + 2\right)e^{-x} + (-1)^{n-2} C_n^2 2e^{-x} =$$
$$= (-1)^n e^{-x} \left(\left(x^2 + 2x + 2\right) - n\left(2x + 2\right) + n\left(n - 1\right)\right) = (-1)^n e^{-x} \left(x^2 - 2\left(n - 1\right)x + \left(n^2 - 3n + 2\right)\right)$$

1206 $y = e^x \cos x$

$$y' = e^x \cos x - e^x \sin x = \sqrt{2}e^x \cos\left(x + \frac{\pi}{4}\right)$$

$$y'' = \sqrt{2}e^x \cos\left(x + \frac{\pi}{4}\right) - \sqrt{2}e^x \sin\left(x + \frac{\pi}{4}\right) = 2e^x \cos\left(x + \frac{\pi}{2}\right)$$

..

$$y^{(n)} = 2^{n/2} e^x \cos\left(x + \frac{\pi n}{4}\right)$$