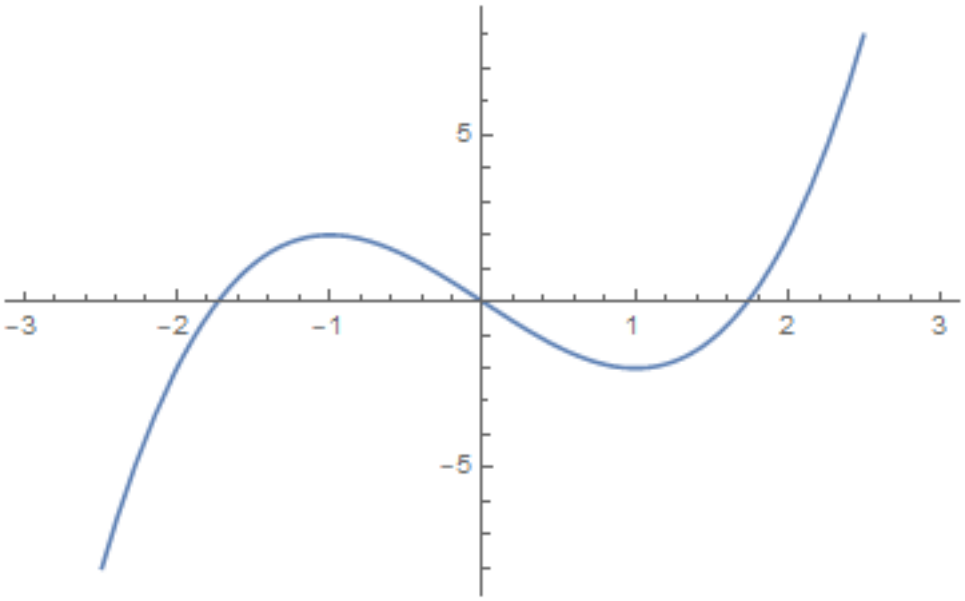
Занятие 12

Исследование функций, заданных явно. Построение графиков

Работа в аудитории

1) $y = x^3 - 3x$



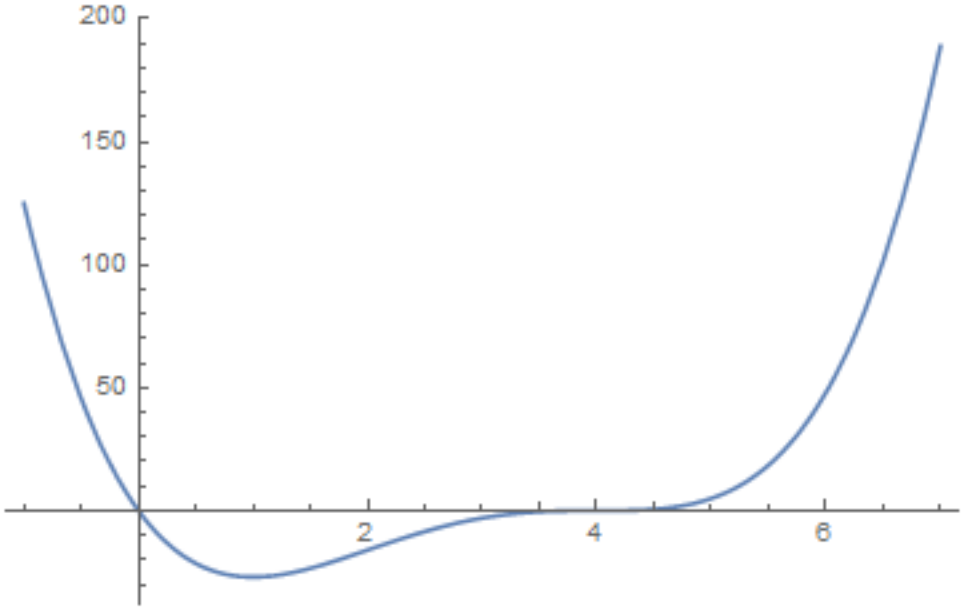
$y' = 3\left(x^2 - 1\right)$

$y'' = 6x$

2) $y = x\left(x-4\right)^3$

$y' = 4\left(x-4\right)^3\left(x-1\right)$

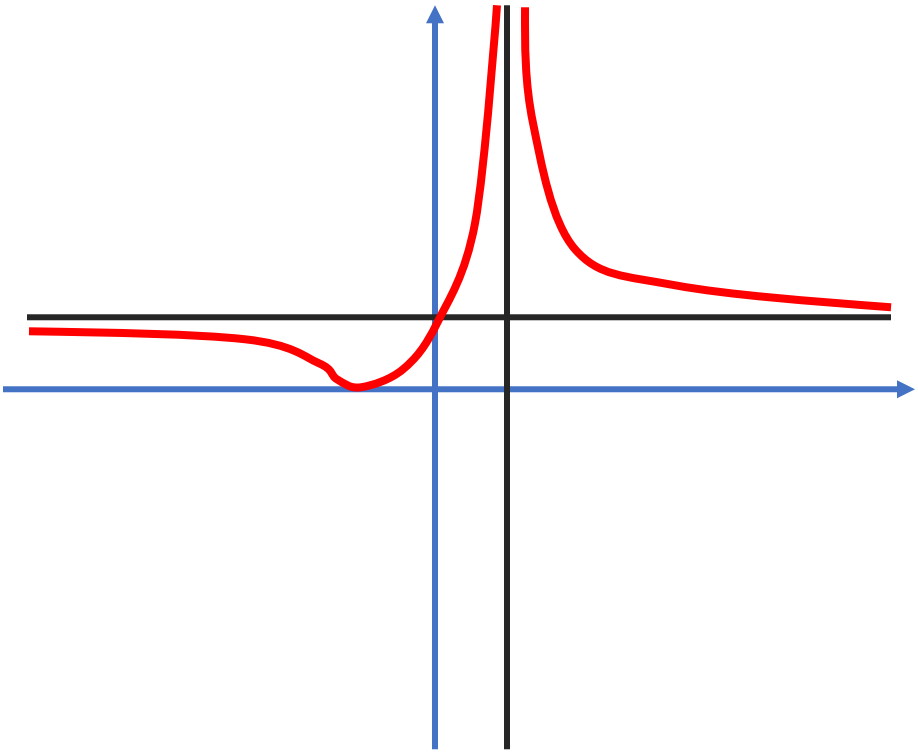
$y'' = 12\left(x-4\right)\left(x-2\right)$



3) 1478 $y=\left(\frac{1+x}{1-x}\right)^4$

$$y'=8\left(\frac{1+x}{1-x}\right)^3\frac{1}{(1-x)^2}=8\frac{(1+x)^3}{(1-x)^5}$$

$$y''=8\left(3\frac{(1+x)^2}{(1-x)^5}+5\frac{(1+x)^3}{(1-x)^6}\right)=8\frac{(1+x)^2}{(1-x)^6}\left(3(1-x)+5(1+x)\right)=16(4+x)\frac{(1+x)^2}{(1-x)^6}$$



4)1481 $y=\dfrac{(x+1)^3}{(x-1)^2}$

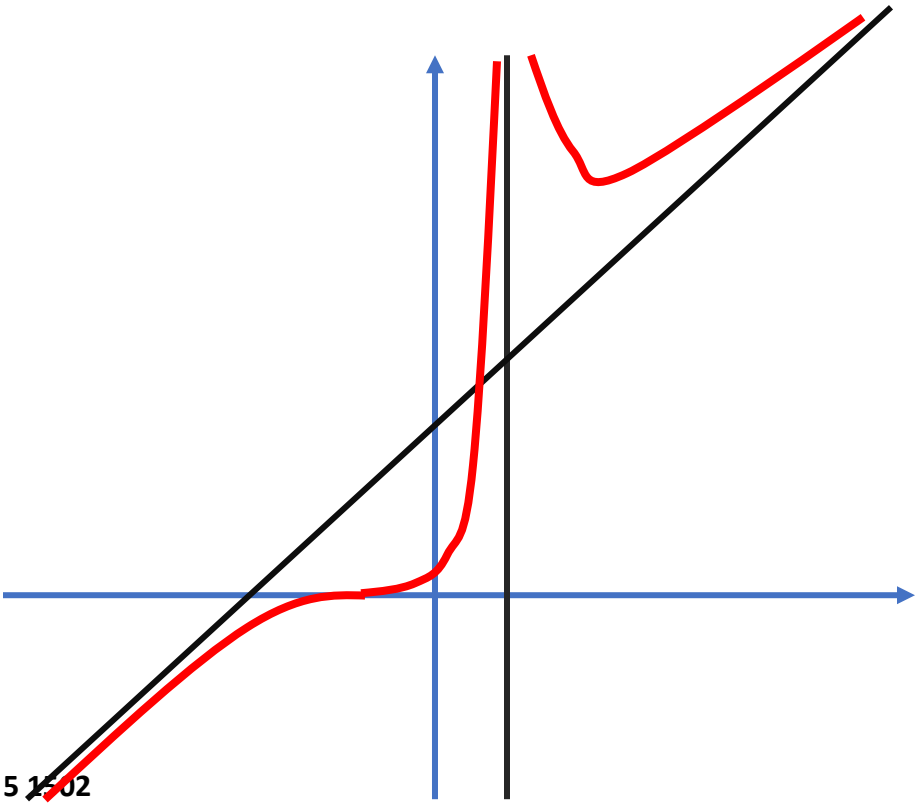
$$y'=3\frac{(x+1)^2}{(x-1)^2}-2\frac{(x+1)^3}{(x-1)^3}=\frac{(x+1)^2}{(x-1)^3}\big(3(x-1)-2(x+1)\big)=\frac{(x+1)^2}{(x-1)^3}(x-5)$$

$$y''=2\frac{(x+1)}{(x-1)^3}(x-5)-3\frac{(x+1)^2}{(x-1)^4}(x-5)+\frac{(x+1)^2}{(x-1)^3}=$$

$$=\frac{(x+1)}{(x-1)^4}\big(2(x-1)(x-5)-3(x+1)(x-5)+(x^2-1)\big)=24\frac{(x+1)}{(x-1)^4}$$

$$y=\frac{(x+1)^3}{(x-1)^2}=x\frac{\left(1+\frac{1}{x}\right)^3}{\left(1-\frac{1}{x}\right)^2}=x\left(1+\frac{3}{x}+\frac{3}{x^2}+o\left(\frac{1}{x^2}\right)\right)\left(1+\frac{2}{x}+\frac{3}{x^2}+o\left(\frac{1}{x^2}\right)\right)=$$

$$=x\left(1+\frac{5}{x}+\frac{12}{x^2}+o\left(\frac{1}{x^2}\right)\right)=x+5+\frac{12}{x}+o\left(\frac{1}{x}\right)$$



5 15.02

$y=\sin x\sin 3x$

$$y = \sin x \sin 3x = \frac{1}{2}\left(\cos 2x - \cos 4x\right)$$

$$y' = -\sin 2x + 2\sin 4x = 2\sin 2x\left(4\cos 2x - 1\right)$$

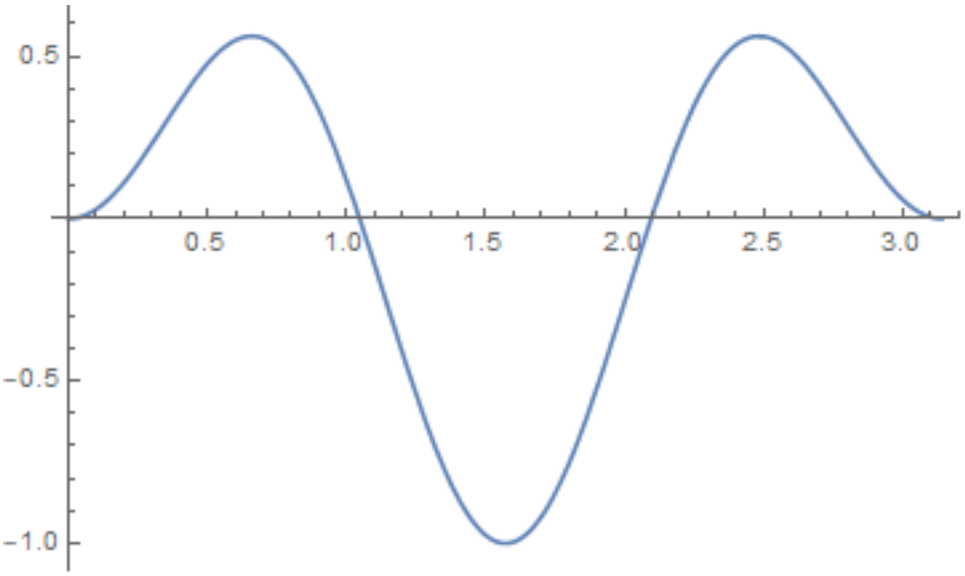
$$0,\ \frac{\pi}{2},\ \pi;\ \frac{1}{2}\arccos\frac{1}{4} \approx \frac{\pi}{5},\ \pi - \frac{1}{2}\arccos\frac{1}{4} \approx \frac{4\pi}{5}$$

$$y'' = -2\cos 2x + 8\cos 4x = 4\left(8\cos^2 2x - \cos 2x - 4\right)$$

$$\frac{1 \pm \sqrt{129}}{16}$$

$$\frac{1}{2}\arccos\frac{1+\sqrt{129}}{16} = \frac{\pi}{10} \quad \pi - \frac{1}{2}\arccos\frac{1+\sqrt{129}}{16} = \frac{9\pi}{10}$$

$$\frac{1}{2}\arccos\frac{1-\sqrt{129}}{16} \approx \frac{\pi}{3} \quad \pi - \frac{1}{2}\arccos\frac{1-\sqrt{129}}{16} \approx \frac{2\pi}{3}$$

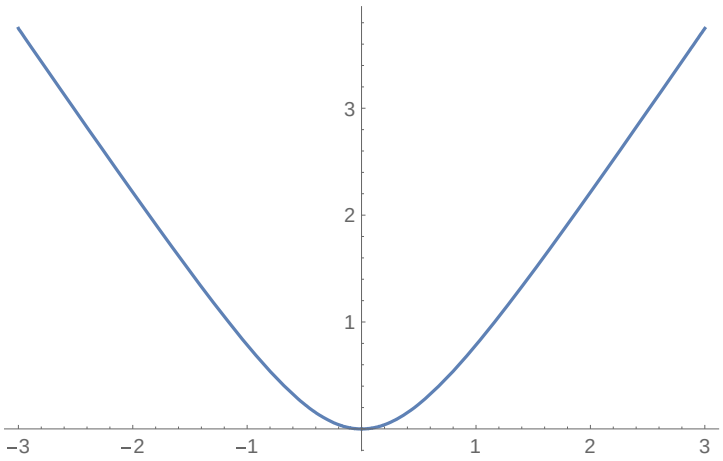


6) 1518 $y = x\,\mathrm{arctg}\,x$

$$y = x\,\mathrm{arctg}\,x$$

$$y' = \mathrm{arctg}\,x + \frac{x}{x^2+1}$$

$$y'' = \frac{2}{x^2+1} - \frac{2x^2}{\left(x^2+1\right)^2} = \frac{2}{\left(x^2+1\right)^2}$$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1
2
3